A Dispersion Model for Coupled Microstrips

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Abstract —The frequency-dependent propagation characteristics of symmetrical, nonsymmetrical, and multiple coupled microstrips are evaluated by utilizing a directly coupled parallel-plate ideal waveguide model. The closed-form expressions for the frequency-dependent parameters of this proposed semi-empirical utility model are derived in terms of the quasi-static parameters of the coupled microstrip structure. These model parameters are then used to evaluate the frequency-dependent propagation characteristics, including the normal-mode effective dielectric constants and impedances of the coupled microstrips. These results are found to be in good agreement with all the published experimental results and the numerically computed values for symmetrical and nonsymmetrical coupled microstrips. The model should be useful in the computer-aided design of coupled microstrip structures at higher frequencies where the dispersion effects become important.

I. Introduction

A CONSIDERABLE amount of work has been done in recent years on the numerical computation of the dispersion characteristics of symmetrical and nonsymmetrical coupled microstrips. Some of these techniques lead to fairly accurate results including, for example, the results obtained by Jansen [1], [2] for coupled symmetrical and nonsymmetrical lines. In addition, several empirical expressions and useful models have been proposed for single and coupled pair of identical microstrips. These include the closed-form expressions for the dispersion characteristics of single and symmetrical coupled microstrips [3], [4] and the LSE, the coupled TEM–TE, and the ideal waveguide models for single microstrips [5]–[8]. Such models, however, are limited to an extension of the single microstrip results [9], [10], to the case of identical coupled lines where an "effective" mode impedance is substituted for the characteristic impedance of the line. According to this model, the dispersion curves for the even- and the odd-mode effective dielectric constants are obtained by replacing the characteristic impedance in the expression with the effective dielectric constant of the single microstrip by half the even- and twice the odd-mode impedances of the coupled pair, respectively. It is readily shown that this model leads to erroneous results for the two extreme coupling cases of loosely and tightly coupled lines [3], [4]. In addition, this model cannot be easily extended to the case of nonsymmetrical and multiple coupled microstrips.

In this paper, a semi-empirical utility model consisting of equivalent waveguides which can be used to evaluate the frequency-dependent behavior of general symmetrical, nonsymmetrical, and multiple coupled microstrips is presented. The model consists of coupled ideal parallel-plate waveguides filled with corresponding effective dielectric media and is somewhat similar to the one used for single microstrips [7], [8]. The model parameters are derived in terms of the quasi-static self- and mutual capacitances and inductances per unit length of the coupled microstrips. It is then assumed that these parameters are dependent on frequency in manner similar to the case of single microstrip model parameters [5], [7], [8]. This leads to the frequency-dependent expressions for all of the model parameters which are then used to evaluate the normal-mode velocities, impedances, and other characteristics of the coupled line structure. The frequency-dependent behavior of all the normal-mode effective dielectric constants conforms to the accepted notion [11] that their values increase monotonically from a zero frequency value corresponding to the static solutions to an asymptotic value equal to the dielectric constant of the substrate as frequency approaches infinity. The inflection frequency in all the model parameters corresponds to the cutoff frequency of the first higher order mode of the system [5], [7], [8], [11] and it is found by applying the transverse resonance condition to the waveguide model representing the coupled microstrip structure.

The procedure for deriving the model parameters that leads to the frequency-dependent coupled microstrip parameters is exemplified in the next section by considering the nonsymmetrical coupled microstrip case in detail. The symmetrical coupled microstrip case is then considered as a simplified special case where the microstrip widths are equal and the multiple coupled microstrips case represents a generalization of the procedure where all the steps are directly translated to the general situation. It should be noted that reliable closed-form empirical expressions are available only for frequency-dependent even- and odd-mode symmetrical coupled microstrip parameters [4], and such work on the modeling or the closed-form empirical expressions for nonsymmetrical and multiple coupled microstrips has not been reported.

II. The Model

In order to illustrate the procedure for evaluating the model parameters, we consider the case of a coupled pair of lines having different widths. The cross-sectional view of

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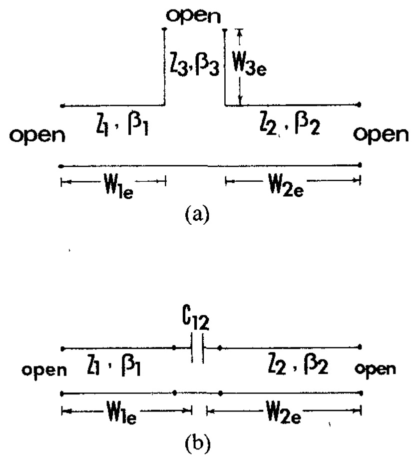

Fig. 1. (a) Coupled microstrips. (b) The proposed waveguide model with electric (———) and magnetic (----) walls.

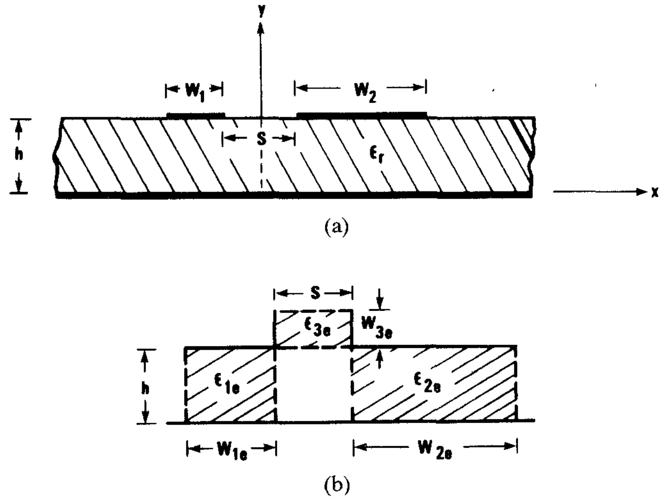

Fig. 2. (a) Equivalent transmission-line circuit in the transverse direction. (b) Same circuit for $W_{3e}\beta_3 << 1$.

this nonsymmetrical coupled microstrip structure and the corresponding ideal waveguide model are shown in Fig. 1. The model consists of three ideal parallel-plate waveguides filled with an effective medium whose plate widths and medium permittivity are derived by imposing the condition that at low frequencies ($f \rightarrow 0$) all of the normal-mode parameters of the model be exactly the same as those of the coupled microstrips [12], that is, at zero frequency $\epsilon_{1e}(0)$, $\epsilon_{2e}(0)$, $\epsilon_{3e}(0)$, $W_{1e}(0)$, $W_{2e}(0)$, and $W_{3e}(0)$ are found by requiring that all the self- and mutual line capacitances, and inductances per unit length of this waveguide model be exactly equal to the corresponding values for the coupled microstrip structure. These quasi-static line constants per unit length for coupled microstrips are the elements of the capacitance and the inductance matrices as given by

$$[C] = \begin{bmatrix} C_{11} & -C_{12} \\ -C_{12} & C_{22} \end{bmatrix}$$

$$[L] = \begin{bmatrix} L_{11} & L_{12} \\ L_{12} & L_{22} \end{bmatrix} = \mu_0 \epsilon_0 \begin{bmatrix} C_{110} & -C_{120} \\ -C_{120} & C_{220} \end{bmatrix}^{-1}. \quad (1)$$

Here, the inductance matrix is expressed in terms of the capacitance matrix of the same coupled microstrip structure without the dielectric medium. Equating the line constants for the model and the coupled microstrips leads to the following expressions for the effective plate widths and dielectric constants of the model:

$$\epsilon_{1e}(0) = \frac{C_{11} - C_{12}}{C_{110} - C_{120}} \quad (2a)$$

$$\epsilon_{2e}(0) = \frac{C_{22} - C_{12}}{C_{220} - C_{120}} \quad (2b)$$

$$\epsilon_{3e}(0) = \frac{C_{12}}{C_{120}} \quad (2c)$$

$$W_{1e}(0) = h(C_{110} - C_{120})/\epsilon_0 \quad (2d)$$

$$W_{2e}(0) = h(C_{220} - C_{120})/\epsilon_0 \quad (2e)$$

$$W_{3e}(0) = sC_{120}/\epsilon_0. \quad (2f)$$

Now, the dispersion characteristics of the coupled microstrip structure can be evaluated from the frequency-dependent behavior of the model parameters as done for the single line case. The effective dielectric constants and widths of each ideal waveguide are assumed to be frequency-dependent in the same manner as other proposed models and empirical formulas [3]–[11]. The variation in both sets of parameters (i.e., the effective plate widths and the dielectric constants) is characterized in terms of an inflection frequency f_p, corresponding to the cutoff frequency of the first higher order mode of the structure [5], [11]. In addition, all of the effective waveguide parameters monotonically approach their asymptotic values as $f \rightarrow \infty$.

The inflection frequency corresponds to the cutoff frequency of the first higher order mode of the system and, following Getsinger and subsequent work, this frequency is seen to correspond to the lowest transverse resonance frequency of the structure. This can be evaluated by considering the two parallel-plate lines 1 and 2 coupled via the third line 3 forming a T-junction as shown in Fig. 2(a). The transverse resonance condition for the structure is found by requiring that the total impedance or admittance of the structure be zero at any convenient plane, and is given by

$$\frac{1}{Z_1}\tan[\beta_1 W_{1e}(0)] + \frac{1}{Z_2 \cot[\beta_2 W_{2e}(0)] + Z_3 \cot[\beta_3 W_{3e}(0)]} = 0 \quad (3)$$

where

$$Z_j = \sqrt{\frac{\mu_0}{\epsilon_0}} \frac{h_j}{\sqrt{\epsilon_{je}(0)}}, \qquad j = 1,2,3, \; h_1 = h_2 = h, \; h_3 = s$$

and

$$\beta_j = \omega\sqrt{\mu_0 \epsilon_0 \epsilon_{je}(0)}, \qquad j = 1,2,3.$$

For small W_{3e} when compared with the widths of the main model lines 1 and 2 and the range of wavelengths over which the characteristics are to be evaluated, the third model line looks only like a frequency-independent capacitor, and for this case the transverse resonance condition becomes

$$\frac{1}{\omega C_{12}} + Z_1 \cot\left[\beta_1 W_{1e}(0)\right] + Z_2 \cot\left[\beta_2 W_{2e}(0)\right] = 0. \quad (4)$$

The lowest order solution of the above equations gives the inflection frequency characterizing the coupled microstrip structure. These equations and similar ones for structures having more then two microstrips are readily solved by trial and error or by writing a simple computer program designed to find the zeros of such expressions. The effective dielectric constants and plate widths of the model are frequency-dependent in accordance with f_p. For the case of small W_{3e} (C_{12} is frequency-independent and f_p is found from (4)), these are given by

$$\epsilon_{je}(f) = \epsilon_r - \frac{\epsilon_r - \epsilon_{je}(0)}{1 + G(f/f_{jp})^2} \quad (5)$$

$$W_{je}(f) = W_j - \frac{W_j - W_{je}(0)}{1 + (f/f_p)^2} \quad (6)$$

with

$$G = 0.6 + 0.009(2\mu_0 h f_p) \quad (7)$$

where $j = 1,2$ refers to the two guides and G is a factor obtained empirically and is taken to be the same as that used in [5] for a single strip with a cutoff frequency f_p for the first higher order mode. The G factor given above has been derived primarily for the alumina substrate and can always be modified such that the results obtained by the model are closer to the experimental or numerically computed values. Note that this modification in the G factor is required perhaps because the coupling to other higher order modes has been excluded and that the inclusion of the effect of these modes on f_p or G becomes quite cumbersome and defeats the basic objective of realizing simple models or expressions that are compatible with computer-aided design procedures.

At higher frequencies or for tightly coupled lines, ϵ_{3e} and W_{3e} are frequency-dependent in the same manner as (6) and (7) with f_p found from the solution of (3).

The frequency-dependent widths and effective dielectric constants of the coupled waveguide model structure characterize the coupled microstrip structure whose self- and mutual line constants per unit length are now frequency-dependent and are given by the parallel-plate formulas as

$$C_{11}(f) = \frac{\epsilon_{1e}(f) W_{1e}(f)}{h} + \frac{\epsilon_{3e}(f) W_{3e}(f)}{s} \quad (8a)$$

$$C_{12}(f) = \frac{\epsilon_{3e} W_{3e}(f)}{s} \quad (8b)$$

$$C_{22}(f) = \frac{\epsilon_{2e}(f) W_{2e}(f)}{h} + \frac{\epsilon_{3e}(f) W_{3e}(f)}{s}. \quad (8c)$$

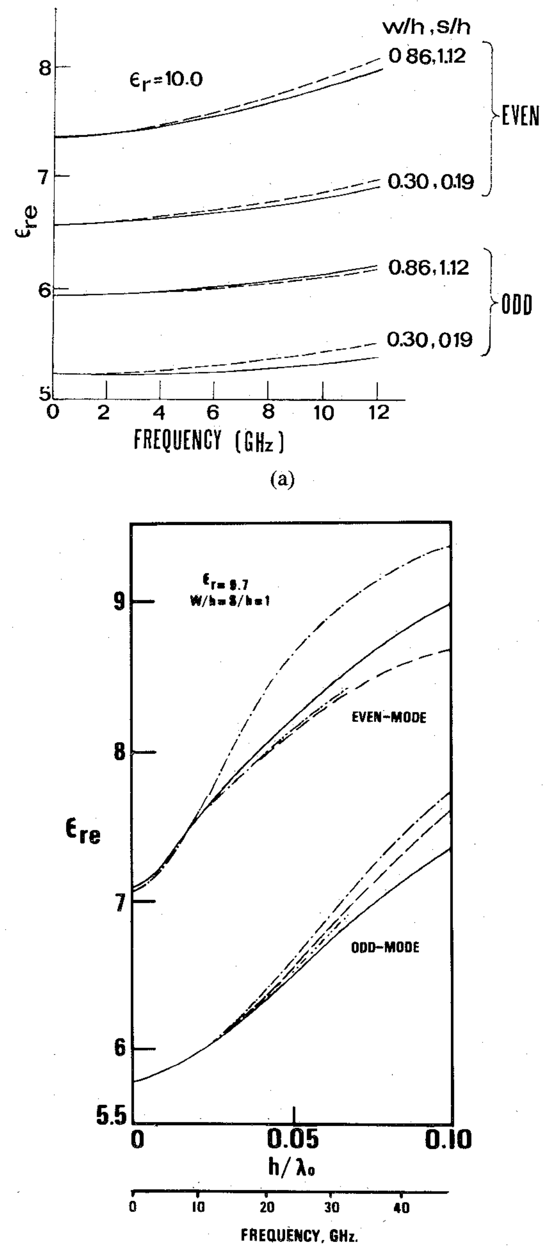

Fig. 3. (a) Effect of dispersion on the effective even- and odd-mode dielectric constants for two typical cases from [9]. ——— This model. ---- From [9]. (b) Frequency-dependence of effective even- and odd-mode dielectric constants for a typical case from [14]. ——— This model. ---- Results computed by Kowalski and Pregla [14]. - - - - - Getsinger's model [9]. - · - · - · - From Kirshning and Jansen [4]

The corresponding values in air medium which are needed for inductance calculations in (1) are given by the same expressions as (9a, b, and c) by replacing $\epsilon_{je}(f)$ by ϵ_0 for all $j = 1,2$, and 3.

These line constants can then be used to find the frequency-dependent normal-mode parameters, i.e., the effective dielectric constants, the model impedances, and the mode voltage ratios which are required in the design of coupled microstrip structures [12], [13]. It is to be noted that, for identical coupled lines, the two normal modes are the even and the odd modes, $W_{1e} = W_{2e}$, $\epsilon_{1e} = \epsilon_{2e}$, and all the calculation steps are significantly simplified.

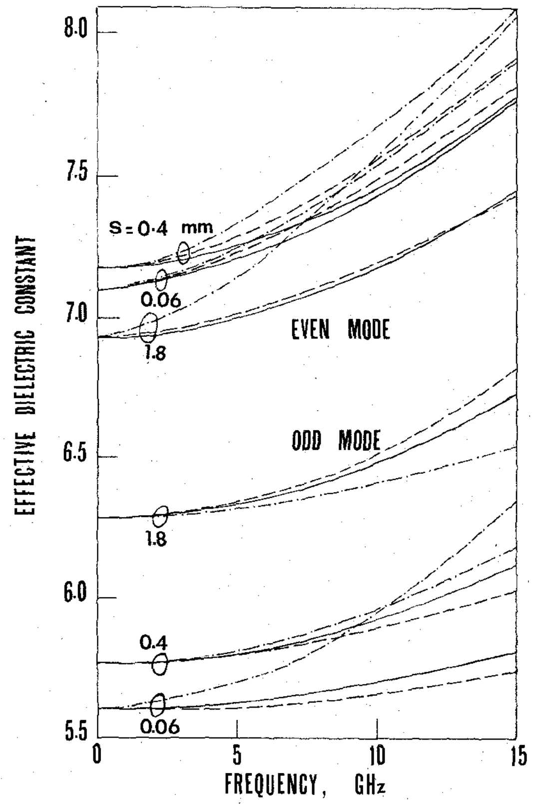

Fig. 4. Frequency-dependent behavior of the even- and odd-mode effective dielectric constants for three typical cases of tightly, moderately, and losely coupled microstrips. $\epsilon_r = 9.7$, $W = 0.6$ mm, $h = 0.635$ mm. —— This model. ---- Numerically computed values from [2]. - · - · - · Computed by using the model in [9].

In general, the procedure for evaluating the frequency-dependent parameters can be summarized as follows.

1) Evaluate the quasi-static capacitance matrices for the structure with and without the dielectric medium. For the case of a coupled pair of symmetrical or nonsymmetrical microstrips, these parameters can be evaluated by utilizing the various reliable closed-form expressions that have been proposed (e.g., [3], [14], [15]). In addition, several accurate efficient computer programs are available for the computation of the quasi-TEM parameters of the multiple coupled microstrip structures (e.g., [16], [17]).

2) Find the inflection frequency f_p for the model by solving for the lowest resonance frequency of the corresponding transverse resonance circuit (e.g., (4), Fig. 2(b)). The transverse resonance equation has only one unknown which can be readily found graphically or by trial and error or numerically by writing a simple program designed to find the f_p for which the resonance equation is satisfied.

3) Find all the effective widths $W_{je}(f)$ and the dielectric constants at any frequency of interest for the coupled ideal waveguide model by using (3)–(5) for coupled microstrips or corresponding equations for multiple coupled microstrip systems.

4) Find the characterizing frequency-dependent equivalent capacitance and inductance matrix elements in terms of $W_{je}(f)$ and $\epsilon_{je}(f)$ which lead to the frequency-dependent normal-mode parameters of the coupled line systems [12], [13], [18].

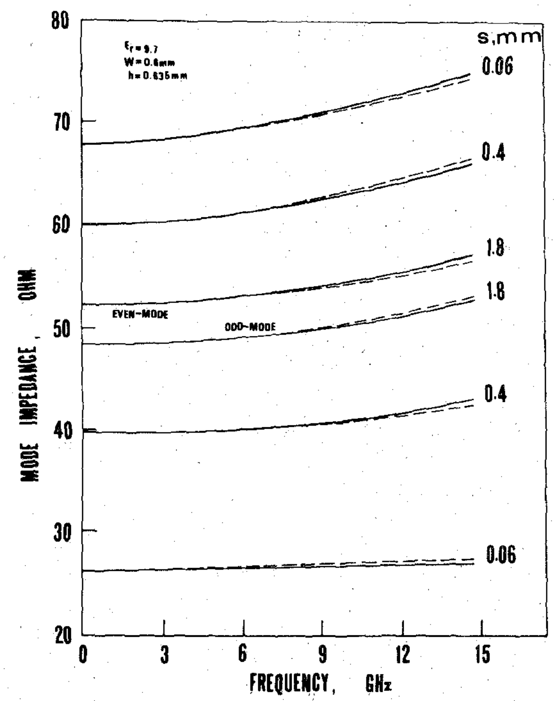

Fig. 5. Effect of dispersion on the even- and odd-mode impedances of symmetrical coupled microstrips. —— This model. ---- Numerically computed values from [2].

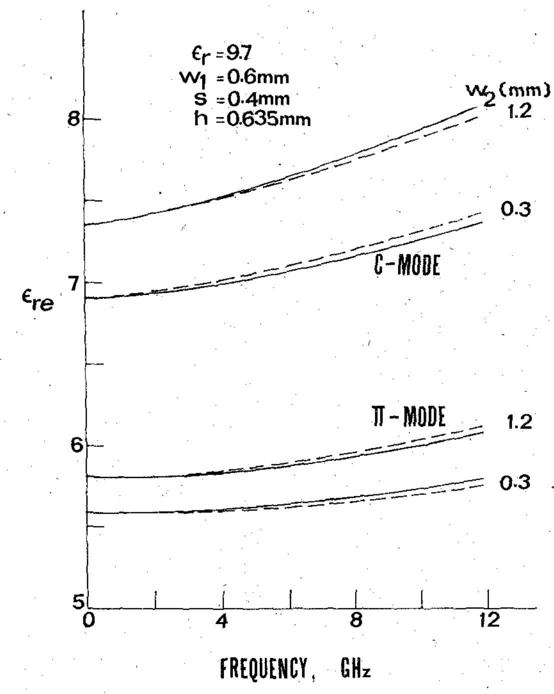

Fig. 6. Effect of dispersion on the normal-mode effective dielectric constants for nonsymmetrical coupled microstrips. —— This model. ---- Numerically computed values from [1].

III. Results

Some results for the frequency-dependent propagation characteristics of coupled microstrip structures are shown in Figs. 3–7, these typical cases considered include symmetrical coupled microstrips (Figs. 3–5), nonsymmetrical coupled microstrips (Fig. 6), as well as a symmetrical three-line structure (Fig. 7). The quasi-static or low-

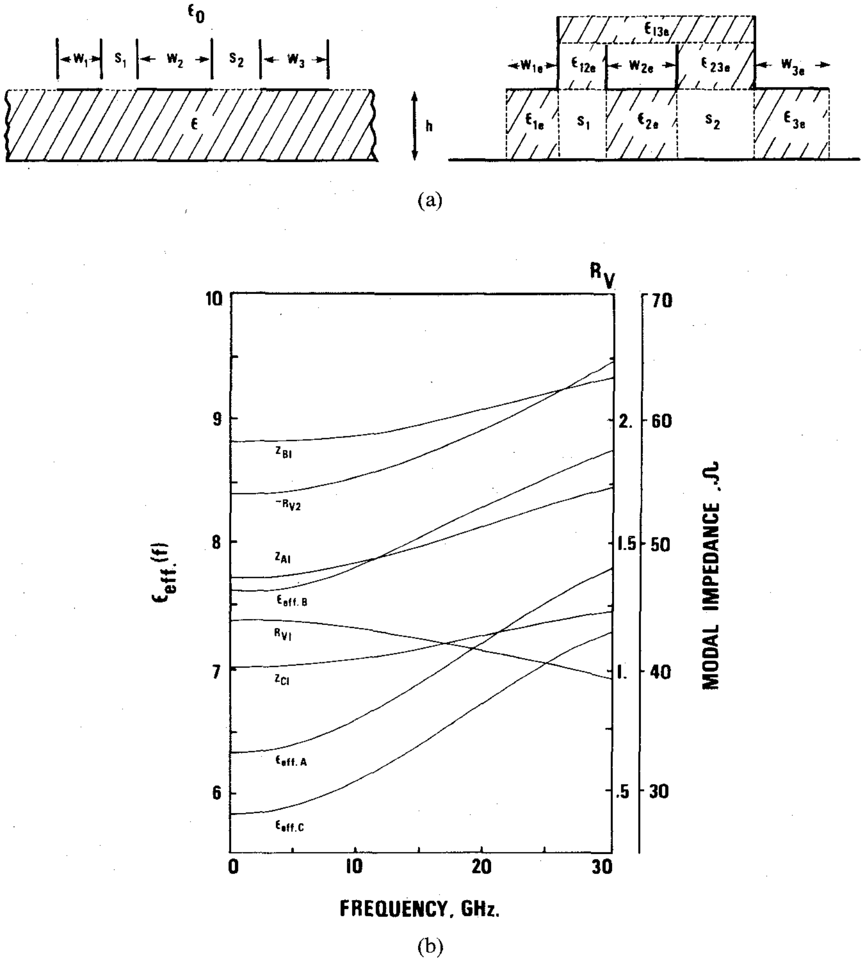

Fig. 7. (a) A three-line structure and the corresponding waveguide model. (b) The normal-mode parameters of a symmetrical three-line structure as a function of frequency. $W_1 = W_2 = W_3 = S_1 = S_2 = h$ and $\epsilon = 10\ \epsilon_0$.

frequency parameters for symmetrical coupled lines were evaluated by using the zero-frequency even- and odd-mode data given in [9], [2], and [14] so that comparison of the results obtained from the model with the corresponding published work can be more meaningful. For the case of nonsymmetrical lines, the model parameters cannot be deduced from [1]. They can, of course, be easily evaluated by using any capacitance calculation technique or program such as the one used in [16] or one of the recently proposed closed-form expressions for nonsymmetrical coupled microstrips [15]. In addition, accurate quasi-static normal-mode parameters for general multiple coupled microstrips including nonsymmetrical coupled pairs of lines and symmetrical three-line structures can be obtained from [17]. For the examples considered in Fig. 6, the zero-frequency effective dielectric constant for the two normal modes given in [1] was close to the values calculated by the techniques given in both [16] and [17].

Figs. 3 and 4 show the frequency-dependent even- and odd-mode effective dielectric constants for some symmetrical coupled microstrips. The two cases considered in Fig. 3(a) are the ones for which the results obtained from the model in [9] were found to be excellent agreement with experimental results. In Fig. 3(b), the computed results from this model are compared with those based on a hybrid-mode analysis [14] and recently published reliable closed-form expressions for symmetrical coupled lines [4] together with the results calculated from the model in [9]. The results based on the model presented here are in good agreement with these given in [4] and [14] up to at least 30 GHz for this case. The three cases shown in Fig. 4 are chosen to demonstrate the effect of coupling or spacing between the lines on the dispersion characteristics, and it is seen that the model used in [9] may indeed lead to erroneous results for the even-mode effective dielectric constant for loosely coupled and the odd-mode effective value for the tightly coupled microstrips. Fig. 5 shows the variation of the even- and odd-mode characteristic impedances as a function of frequency together with the numerically computed values based on the voltage over current definition [2], which is in conformity with impedance definitions for the transmission-line model. Fig. 6 shows the effective dielectric constants for the two normal modes of propagation for nonsymmetrical coupled microstrips together with the numerically computed results from [1]. The normal-mode parameters for a symmetrical three-line structure [18] are shown in Fig. 7. The model for this is shown in Fig. 7(a) and it consists of six parallel-plate ideal waveguides as defined from the quasi-static data computed by using the numerical techniques and results in [17]. All of the results presented above indicate that the waveguide model is indeed a valid model in that the results obtained from the model are in good agreement with the available numerical and experimental results.

IV. Conclusions

A simple procedure to evaluate the frequency-dependent propagation characteristics of coupled microstrip structures has been presented. The semi-empirical utility model which has been proposed here is particularily suited for the computer-aided design of coupled microstrip circuits since it does not require any special numerical techniques normally associated with the hybrid-mode analysis of planar structures. The results for the frequency-dependent normal-mode parameters of coupled microstrips are obtained by utilizing the closed-form expressions presented here together with the known empirical expressions or numerical results for quasi-static parameters of the coupled microstrip system. The model presented is a general one and it has been applied to the case of a coupled pair as well as multiple coupled microstrip structures. It should be noted that this model is by no means an exact or unique mathematical model and has room for considerable further improvement as more reliable and accurate results become available. The expressions for the model parameters including the inflection frequency and the factor G can be modified for a better fit as these accurate numerical and experimental results become available particularly at higher frequencies for symmetrical, nonsymmetrical, as well as multiple coupled microstrip structures.

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